MCV4U Practice Exam: Calculus Component

Part A: Multiple Choice

For questions 1 to 10, select the best answer.

1. Which expression represents the first principles definition of the derivative of $f(x)$?

A $\dfrac{f(x+h)-f(x)}{h}$

B $\lim\limits_{h\to 0}\dfrac{f(x-h)+f(x)}{h}$

C $\lim\limits_{h\to 0}\dfrac{f(x-h)-f(x)}{h}$

D $f'(x)$

2. Evaluate $\lim\limits_{x\to 3}\dfrac{9-x^2}{x-3}$.

A 0 **B** ∞

C 6 **D** -6

3. State the intervals of increase for $y=f(x)$ given $f'(x)=-3x(x-2)(x+1)$.

A $-1<x<0$ and $x>2$

B $-1<x<2$

C $x<-1$ and $0<x<2$

D $x<-1$ and $x>2$

4. Which is the slope of the tangent to the curve $y=4x^3$ at $x=2$?

A 48 **B** 12

C 4 **D** 32

5. Which expression is equivalent to $\lim\limits_{h\to 0}\dfrac{(x+h)^{100}-x^{100}}{h}$?

A x^{100}

B $100x^{99}$

C 0

D ∞

6. Which must be true for a minimum to occur at $x=a$ on $y=f(x)$?

A $f'(a)>0$

B $f'(a)<0$

C $f''(a)>0$

D $f''(a)<0$

7. Which must be true for a critical point to occur at $x=a$ on $y=f(x)$?

A $f'(a)=0$

B $f''(a)=0$

C $f''(a)>0$

D $f''(a)<0$

8. Which is the derivative of $y=\dfrac{-3}{x^2}$?

A $y'=\dfrac{3}{2x}$

B $y'=-\dfrac{6}{x^3}$

C $y'=\dfrac{6}{x^3}$

D $y'=-6x^4$

9. Which is the derivative of $y=2\sqrt{x}$?

A $y'=\dfrac{1}{2\sqrt{x}}$

B $y'=\dfrac{1}{\sqrt{x}}$

C $y'=\dfrac{1}{2}\sqrt{x}$

D $y'=x^{-\frac{3}{2}}$

10. Which is the derivative of $f(x)=8^x$?

A 8^x

B $x\ln 8$

C $8^x\ln 8$

D $\ln 8$

Part B: Extended Response

Show all the steps of each solution.

11. Differentiate.

a) $y = (2x^2 - 1)^3(x^4 + 3)^5$

b) $f(x) = \dfrac{6x + 5}{\sqrt{7 - 3x^2}}$

c) $y = \sin(x^3)\cos^3 x$

d) $h(x) = \dfrac{x^2}{e^{3 - 4x}}$

12. Evaluate each limit, if it exists. If it does not exist, explain why.

a) $\lim\limits_{x \to 0} \dfrac{\sqrt{16 - x} - 4}{x}$

b) $\lim\limits_{x \to 2} \dfrac{3x^2 - 7x + 2}{2x^2 - x - 6}$

13. Where is this function discontinuous? Justify your answer.

$$f(x) = \begin{cases} -(x + 2)^2 + 1 & \text{if } x \leq 2 \\ x + 1 & \text{if } -2 < x \leq 3 \\ (x - 3)^2 - 1 & \text{if } x > 3 \end{cases}$$

14. Use first principles to determine the derivative of $f(x) = \dfrac{2x}{x - 3}$.

15. Determine the coordinates of the points on the graph of $y = \dfrac{2x^2}{3x - 1}$ at which the slope of the tangent is 0.

16. Consider the function $f(x) = \dfrac{-3}{x^2 - 4}$.

a) Determine the domain, the intercepts, and the equations of the asymptotes.

b) Determine the local extrema and the intervals of increase and decrease.

c) Determine the coordinates of the point(s) of inflection and the intervals of concavity.

17. Determine the maximum volume of a square-based box with an open top that can be constructed with 3600 cm^2 of cardboard.

18. A store sells 380 frozen yogurt cakes per week at a price of \$12.50 each. A market survey indicates that for each \$0.25 decrease in price, five more cakes will be sold each week.

a) Write the demand function.

b) Write the revenue function.

c) Determine the marginal revenue.

d) For what price is the marginal revenue zero? Interpret the meaning of this value.

19. An oceanographer measured an ocean wave during a storm. The vertical displacement, h, of the wave, in metres, can be modelled by $h(t) = 0.8\cos t + 0.5\sin 2t$, where t is the time in seconds.

a) Determine the vertical displacement of the wave when the velocity is 0.8 m/s.

b) Determine the maximum velocity of the wave and when it first occurs.

c) When does the wave first change from a "hill" to a "trough"? Explain.

MCV4U Practice Exam: Vector Component

Part A: Multiple Choice

For questions 1 to 12, select the best answer.

1. Which is *not* an example of a vector?
A force **B** displacement
C speed **D** velocity

2. Which statement is *always* true?
A Parallel vectors have the same direction.
B Equivalent vectors have the same magnitude.
C Vectors are subtracted by adding the opposite.
D The resultant of two opposite vectors is the zero vector.

3. Given vectors $\vec{a}$ and $\vec{b}$ and scalar k, which is meaningless?
A $k\vec{a}$ **B** $\vec{a} \times b$
C $\vec{a} \cdot \vec{b}$ **D** $\vec{a}\ \vec{b}$

4. In three space, which is the definition of skew lines?
A Lines that intersect in a point.
B Non-parallel, non-intersecting lines.
C Lines that are perpendicular.
D Lines that are parallel.

5. Which vector equation represents a line through A(4, 3, 1) and B(−2, 1, 0)?
A $[x, y, z] = [4, 3, 1] + t[-2, 1, 0]$
B $[x, y, z] = [4, 3, 1] + t[2, 4, 1]$
C $[x, y, z] = [-2, 1, 0] + t[-6, -2, 1]$
D $[x, y, z] = [4, 3, 1] + t[6, 2, 1]$

6. Which expression is equivalent to $2(3\vec{i} - \vec{j} + \vec{k}) - (\vec{i} + 2\vec{k})$?
A [5, 2, 0] **B** $[5\vec{i} - 2\vec{j}]$
C [5, 2, 4] **D** $5\vec{i} - 2\vec{j}$

7. Which statement is *not* true?
A A line in two-space can be represented by a vector equation.
B A line in three-space can be represented by a scalar equation.
C A plane in three-space can be represented by a scalar equation.
D A plane in three-space can be represented by a vector equation.

8. Which scalar equation represents the same line as $[x, y] = [2, -2] + t[3, -1]$?
A $3x - y - 8 = 0$ **B** $x + 3y + 4 = 0$
C $3x + y - 4 = 0$ **D** $x - 3y + 8 = 0$

9. Which expression is meaningless?
A $\vec{a} \times \vec{b} \times \vec{c}$ **C** $\vec{a} \times \vec{b} \cdot \vec{c}$
B $\vec{a} \cdot \vec{b} \cdot \vec{c}$ **D** $(\vec{a} \cdot \vec{b}) \times \vec{c}$

10. Which statement is *not* correct?
A $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
B $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
C $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
D $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

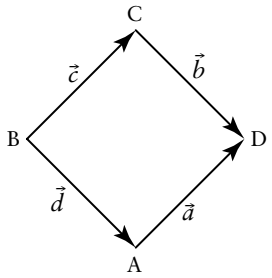
11. Which expression represents a unit vector in the same direction as [1, 2, −1]?
A [1, 1, 1] **B** $\frac{1}{\sqrt{6}}[1, 2, -1]$
C [1, 0, 0] **D** $\frac{1}{2}[1, 2, -1]$

12. Which statement best describes π_1 and π_2?
π_1: $2x - y + 3z - 4 = 0$
π_2: $4x - 2y + 6z - 7 = 0$
A π_1 and π_2 are parallel.
B π_1 and π_2 intersect in a single point.
C π_1 and π_2 are parallel and coincident.
D π_1 and π_2 are parallel and distinct.

Part B: Extended Response

Show all the steps of each solution.

13. Consider this diagram.



a) Name a vector that is equivalent to $\vec{a} - \vec{b}$.

b) Name a vector that is equivalent to $-\vec{b} - \vec{a}$.

14. The vertices of a triangle are P(−2, 3, 4), Q(3, −1, 1), and R(1, −2, −1).

a) Verify that △PQR is a right triangle.

b) Determine the area of △PQR.

c) Determine the coordinates of S(x, y, z) such that PQRS is a rectangle.

15. An airplane is headed N25°E with a constant velocity of 880 km/h. The plane encounters a wind blowing from S75°W at 65 km/h. Determine the resultant velocity of the plane.

16. A crate with mass 20 kg is suspended from a crane by two chains that make angles of 50° and 35° to the horizontal. Determine the tension in each chain.

17. Consider the vectors $\vec{u} = [-5, 1, -1]$ and $\vec{v} = [2, 4, -3]$.

a) Determine $\text{proj}_{\vec{u}}\,\vec{v}$.

b) Determine $\left|\text{proj}_{\vec{u}}\,\vec{v}\right|$.

18. A force $\vec{F} = [200, 600, 400]$, measured in newtons, acts on an object. The displacement of the object, in metres, is defined by $\vec{d} = [2, 1, 10]$.

a) Determine the work done in the direction of travel.

b) Determine the work done against gravity, which is a force in the direction of the negative z-axis.

19. Determine the equation of a plane that contains the line $[x, y, z] = [1, -2, 3] + t[4, 3, -5]$ and is parallel to the line $[x, y, z] = [1, 0, 9] + t[3, -2, 8]$.

20. Determine the intersection of the planes.
π_1: $3x - y + 4z - 1 = 0$
π_2: $x + 2y + z + 7 = 0$

21. Determine the intersection of these planes. Describe the solution geometrically.
π_1: $x + 3y + 2z - 5 = 0$
π_2: $2x - y - 4z - 4 = 0$
π_3: $4x - 3y + z + 3 = 0$